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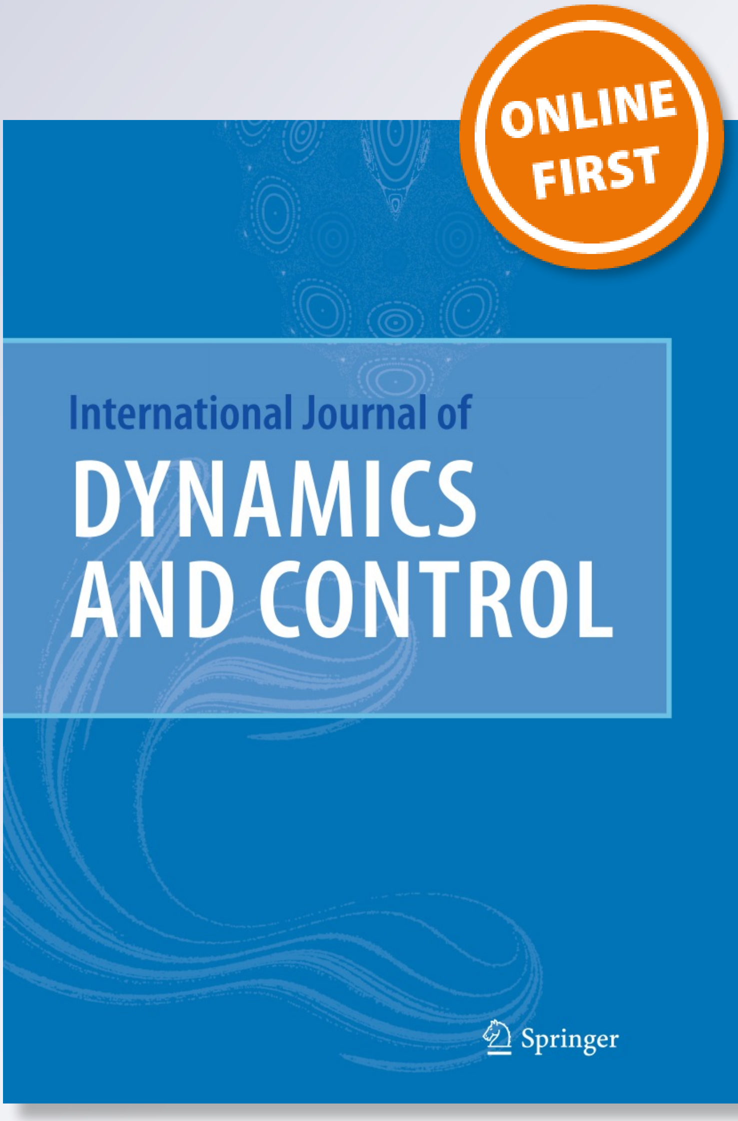

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An adaptive super-twisting sliding mode algorithm for robust control of a biotechnological process

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Abstract

In this paper, a robust super-twisting sliding mode control with adaptive tuning law is developed for a nonlinear biotechnological process, which takes place inside a continuous stirred tank bioreactor. A super-twisting algorithm (STA) is firstly designed to obtain high robustness as well as preserve fast convergence with high accuracy. The benefit of this approach is that its design procedure is independent of the prior knowledge of the bound value of the uncertainties and perturbations. However, the STA has a drawback that provides a chattering in the control loop. In order to overcome this drawback, an adaptive tuning algorithm is developed to adjust the STA control law without frequency switching and alleviate the undesired chattering phenomenon. Then, the robustness can be achieved despite the existence of the unknown uncertainties and external perturbations for the nonlinear process. In addition, a formal proof of the global uniform asymptotic stability based on Lyapunov criterion of the closed-loop process is derived. Several simulation results show that the proposed adaptive super-twisting algorithm guarantees the performance of the STA under external disturbance and parametric uncertainty with less chattering and illustrate the overall performance improvements.

Keywords Nonlinear biotechnological process · Super-twisting sliding mode control · Adaptive control · Stability

1 Introduction

1.1 Background and motivations

In the process industry, continuous bioreactors are largely used for the production or degradation of a wide kind of biological composites counting intracellular production of polyhyroxybutyrate, yeast growth, waste treatment and anaerobic digestion [1]. However, for numerous reasons, the control of bioreactors presents significant challenges. Biological processes are strongly nonlinear and their parameters are highly uncertain [1,2]. These systems include different biological and physicochemical reactions, combined with the uncertain feedstock conditions, time lags and unreliable measuring devices associated with high costs [3]. Nevertheless, an accurate mathematical model is required to design an advanced and robust control for monitoring bioreactors. As documented in the literature, the ADM1 model, which involves in 38 nonlinear differential and algebraic equations system, describes the bioreactors behavior accurately [3,4]. However, this model prohibited the application of further control strategies due to its complexity. Another suitable model for control is the called one-stage [5,6], which contains

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only of 2 ordinary differential equations. It has demonstrated its practicality for bioprocess control in many research works [5,6], and it is used in this paper as well. These reasons increase the necessity of designing new estimation and control schemes to improve the efficiency of the operation process.

1.2 Literature survey

Several control methods have been proposed in the literature in order to overcome the difficulties previously discussed, such as the adaptive [5–7], fuzzy and neural networks methods [8–10]. The adaptive control provides a high performance for the bioprocesses. Nevertheless, the occurrence of large and abrupt changes in the process parameters might lead to the failure of the control process. Moreover, the application of fuzzy and neural strategies can lead to better performance of the bioprocess and can offer good responses. However, the major problem of these strategies is the high computational burden, which leads to increasing the controllers cost.

During last years, the sliding mode control (SMC) algorithm has gained a rapid expansion due to its advantages such as robustness, compact implementation, controller order reduction, low computational complexity and insensitivity to parameter changes. Besides, the SMC has been extensively accepted as an effective technique for the control of uncertain non-linear systems in electrical and mechanical processes [11–14], moreover, its applicability has been recently extended to chemical processes [15]. Furthermore, some attempts for the implementation of the SMC [16,17] and its combination with model-reference adaptive control laws [18,19] are suggested for bioprocesses. These strategies offer good performance in the occurrence of parameter uncertainties and external disturbances.

The main challenge in the practical application of the SMC techniques is the suitable choice of discontinuous control parameters. A discontinuous control law with high gain parameters implies more accuracy and more robustness in the system. However, higher coefficients may produce a larger chattering phenomenon and high switching frequencies in the output of the system, which are the main drawbacks of SMC techniques [20,21]. In order to reduce the chattering phenomenon, Levant suggested in [22] a new SMC algorithm, which is widely known as a super-twisting algorithm (STA). The STA is one of the most powerful second-order continuous sliding mode control algorithms that handle systems with a relative degree equal to one. It produces a continuous control function, which conducts the sliding surface and its derivative to zero in finite time in the existence of the smooth matched disturbances with bounded gradient and known boundary. Meanwhile, the STA comprises a discontinuous function under the integral to decrease chattering. The major drawback of the STA is the requirement of the disturbance gradient boundaries information. Unfortunately, these boundaries cannot be simply assessed in many practical cases. The over-estimation of the disturbance boundary yields to larger than necessary control gains, while designing the super-twisting control law. Therefore, adaptive SMC controllers have been proposed using dynamical gains. The basic idea in such controllers is the adaptation of the gains magnitudes with respect to perturbation /uncertainty effects.

The authors in [23] proposed an adaptive sliding mode controller (first-order) for the control of an electro-pneumatic actuator. Aiming at chattering reduction, the presented controller, which is based on the super-twisting second-order SMC algorithm [22], combines together both approaches, namely, high-order sliding mode control and gain adaptation. This controller needs neither information about the boundaries of the disturbance nor about its gradient.

The authors in [24] proposed an adaptive-gain super twisting control law, which handles the perturbed system dynamics with the additive uncertainty/disturbance of certain class without knowing the boundary [25,26]. The proof is founded on a recently suggested Lyapunov function. Although the presence of the bounded disturbance, the control algorithm constantly conducts the sliding surface and its derivative to zero in finite convergence time.

1.3 Contributions and paper organization

In this paper, a STA is proposed in a first step for the task of controlling of the overall behavior of a given biotechnological process, namely a CSTB. The biotechnological process is described by a two-step reaction scheme based on a second-order nonlinear model, and the control objective is to regulate the concentration of some pollutants at a constant low-level. Furthermore, an adaptation law is introduced in a second step to define suitably the switching gain of the super-twisting algorithm for a non-linear biotechnological process control. The proposed adaptation algorithm consists of designing a dynamically adapted control gains that ensure the establishment, in a finite time, of a real second-order sliding mode. The stability and convergence properties of the resulting non-linear and non-autonomous system are proven through the Lyapunov's method. The main advantages of using the STA with an adaptive algorithm is the enhancement of the system performances, i.e., a high level of regulation accuracy, minimizing the settling time response, increasing the system robustness against external disturbances and model uncertainties. Moreover, the proposed tuning algorithm for the controller gains allows covering a large set of disturbances, with unknown bounds, while minimizing the chattering phenomena faced the application of conventional SMC techniques.

A complete simulation model for the proposed controller is developed with the MATLAB environment. Simulation results confirm the feasibility and performance improvement of the proposed controller at different operating conditions to both conventional SMC and super-twisting controllers. The main contributions of this paper, in summary, are as follows:

1. A STA design procedure is employed to derive the control output of the second-order nonlinear dynamic system, then that it can achieve a high robustness, fast convergence, low steady state error, and globally uniformly asymptotic stability.
2. An adaptive algorithm is proposed to provide high control performance and chattering elimination. The proposed approach does not introduce any additional complexity.
3. In comparison with other control algorithms, such as conventional first-order sliding mode controller and STA, the proposed ASTA provides a superior performance.

The remainder of this paper is organized as follows: in Sect. 2, the dynamical model of the prototype bioprocess is described briefly. Section 3 gives the detail of the proposed super-twisting control approach as well as a detailed analysis of the stability characteristics of the system. The proposed adaptive algorithm for the gain parameters is presented in Sect. 4. The results of a comprehensive simulation using MATLAB are provided in Sect. 5 to evaluate the performances of the proposed control approach. Section 6 concludes the paper.

2 The dynamical model of the prototype biotechnological process

The biotechnological process control is often limited to the regulation of the pH and temperature at constant values that are favorable to the microbial growth. There is, however, no doubt that the control of the biological state variables (biomass, substrates, etc.) can help to increase the performance [1,2]. To develop and apply advanced control strategies for these biological variables, it is obligatory to obtain a useful dynamical model.

By means of a mass balance of the components inside the bioreactor and obeying some modeling rules, a dynamical state-space model of a prototype continuous bioprocess that takes place in a CSTB is defined by the following non-linear system [5,6]:

$$\begin{cases} \dot{X} = \mu(S)X - DX \\ \dot{S} = -K_1\mu(S)X - DS - u \\ e = X - X^* \end{cases} \tag{1}$$

where $X(g/l)$ is the biomass concentration, $S(g/l)$ is the substrate concentration, $D(h^{-1})$ is the dilution rate, K_1 is the yield coefficient, $u(gl^{-1}h^{-1})$ is a control variable defined as the flow rate of the substrate supply to the reactor per unit of volume and $X^*(g/l)$ is the desired concentration of biomass.

For this specific bioprocess, the reaction rate is of the form $\phi(X, S) = \mu(S)X$, with $\mu(S)$ is a non-linear function representing the specific growth rate. Numerous models for the specific growth rate are used in the modeling of bioprocesses [1]. One of the most common models for the specific growth rate is the Monod kinetic model.

$$\mu(S) = \frac{\mu^* S}{(K_M + S)} \tag{2}$$

where μ^* is the maximum specific growth rate and K_M is the Michaelis–Menten constant. In order for the biomass concentration value X to converge to the prescribed set-point value specified by X^*, the control goal for the bioprocess given by Eq. (1) is to obtain a zero error, i.e., $e = 0$.

3 Control design using super-twisting algorithm

This section addresses the development of an efficient strategy for the robust control of the bioprocess behavior defined by the dynamical model given by Eqs. (1) and (2). The control objective is to regulate the concentration of some pollutants in a waste treatment process to a predefined low-level. The proposed control approach is based on the super-twisting control algorithm, which represents one of the most studied algorithms in second-order SMC theory [21–27]. It is characterized by its simplicity of development, robustness against bounded uncertainties as well as its high level of accuracy compared with first-order sliding mode control algorithms [21].

3.1 Super-twisting algorithm

As in general case, the sliding mode control approach usually consists of two steps [12,14]:

– *Step 1* Sliding manifold design: define a fictitious output variable (referred to as the "sliding variable") depending on the measurable output and a certain number of its derivatives, whose vanishing guarantees that the resulting system behavior (i.e. the associated zero-dynamics) meets the desired performance specifications.
– *Step 2* Controller design: define a control action that steers to zero in finite time the sliding variable, despite of model uncertainties and disturbances.

Thus, let us consider the sliding mode manifold defined as follows:

$$\sigma = \dot{e} + \lambda e = \mu S X - D X + \lambda (X - X^*) \tag{3}$$

with λ is a positive constant. Taking the first-time derivative of the Eq. (3) to get:

$$\begin{aligned} \dot{\sigma} &= \frac{\mu^* K_M}{(K_M + S)^2} X + \mu(S)\dot{X} - D\dot{X} + \lambda \dot{X} e) \\ &= \mu'(S) X[-K_1 \mu(S) X - DS] + (\mu(S) - D + \lambda) \\ &\quad \times (\mu(S) - D) + \mu'(S) X u \end{aligned} \tag{4}$$

with

$$\mu'(S) = \frac{\mu^* K_M}{(K_M + S)^2} \tag{5}$$

Equation (4) might be rewritten as:

$$\dot{\sigma} = \Psi + \Gamma u \tag{6}$$

With

$$\begin{cases} \Psi = \mu'(S) X[-K1\mu(S) X - DS] + \\ \qquad (\mu(S) - D + \lambda)(\mu(S) - D) \\ \Gamma = \mu'(S) X \end{cases} \tag{7}$$

The functions Ψ and Γ are supposed to be such that:

$$\begin{cases} \mid \Psi \mid \leq \Psi_M \\ \Gamma_m \leq \mid \Gamma \mid \leq \Gamma_M \end{cases} \tag{8}$$

It is assumed that Ψ_M, Γ_m and Γ_M exist but are not known.

Let us define the linearizing control input $\bar{u}$ as:

$$\bar{u} = \mu'(S) X u = \Gamma u \tag{9}$$

To get:

$$\dot{\sigma} = d(t) + \bar{u} \tag{10}$$

where $d(t)$ $(d(t) = \Psi + \dot{d}(t))$ is a time dependent function introduced to model all uncertainties and external disturbances.

Assumption In this paper it is considered that the perturbation $d(t)$ is Lipschitz and satisfies the following inequality:

$$\mid \dot{d}(t) \mid \leq c \tag{11}$$

Considering the system dynamic Eq. (10) and the assumption Eq. (11), one can use the STA for the stabilization of the system Eq. (9). The advantages of super-twisting controllers are well-known as their designs do not need the derivative of the sliding variable, which is mandatory to find the control law in conventional SMC controllers. In addition, the STA can be implemented to any system that has a relative degree equal to 1 with respect to sliding manifold. In contrast, other algorithms of second-order sliding mode are only applied to a system that has a relative degree equal to 2 with respect to sliding variable using limited controller inputs [21].

The STA algorithm is given as a sum of two components and presented by the behind control law [28,29]:

$$\begin{cases} \bar{u} = -\theta \mid \sigma \mid^{\frac{1}{2}} sign(\sigma) + u_1 \\ \dot{u_1} = -K sign(\sigma) \end{cases} \tag{12}$$

From Eq. (12), it is clearly observed that the STA does not require the estimation of $\dot{\sigma}$. Substituting $\dot{u_1}$ from Eq. (12) in Eq. (10) gives:

$$\begin{cases} \dot{\sigma} = -\theta \mid \sigma \mid^{\frac{1}{2}} sign(\sigma) + u_1 + d \\ \dot{u_1} = -K sign(\sigma) \end{cases} \tag{13}$$

By means of the transformation:

$$\xi = d - K \int_0^t sign(\sigma) d\tau \tag{14}$$

Equation (13) can be expressed as:

$$\begin{cases} \dot{\sigma} = -\theta \mid \sigma \mid^{\frac{1}{2}} sign(\sigma) + \xi \\ \dot{\xi} = -K sign(\sigma) + d \end{cases} \tag{15}$$

3.2 Stability analysis

Considering Eq. (15), this system may be considered as a non-autonomous system. Thus its stability analysis requires the introduction of the basic concepts of uniform stability in the system characteristics analysis [12]. The uniform stability of the non-autonomous system Eq. (15) may be shown using the following theorem.

Theorem 1 *The system given by Eqs. (10) and (11) under the control input (Eq. (12)) is globally uniformly asymptotically stable if the following conditions are hold:*

$$\begin{cases} p_2 > 1,\, p_3 = \dfrac{1+p_2}{c} and p_1 > \dfrac{cp_2^2}{p_2 - 1} \\ \theta = \dfrac{2p_3 + p_2 p_1 + 2p_3(p_3 - 2p_2)c}{2p_1 p_3 - p_2^2} \\ \quad k = \dfrac{p_1 + p_2\theta}{2p_3} \end{cases} \tag{16}$$

Proof Let us define a new variable state ξ:

$$\xi = [\mid \sigma \mid^{\frac{1}{2}} sign(\sigma), \xi]^T = [\xi_1, \xi_2]^T \tag{17}$$

and let us consider the following candidate Lyapunov function:

$$V = \frac{1}{2}\xi^T P \xi, \qquad P = \begin{bmatrix} p_1 & -p_2 \\ -p_2 & p_3 \end{bmatrix} \tag{18}$$

where p_1, p_2 and p_3 are positive constants. This Lyapunov function is differentiable almost everywhere.

If the parameters p_i are chosen so that the inequalities $p_1 > 0$, $p_3 > 0$ and $p_1 p_3 - p_2^2 > 0$ are satisfied, then $V(\xi)$ is positive definite and radially unbounded, or:

$$\frac{1}{2}\lambda_{min}[P]||\xi||_2^2 \leq V(\xi) \leq \frac{1}{2}\lambda_{max}[P]||\xi||_2^2 \tag{19}$$

where $\lambda_{min}[\Delta]$ represents the operation of taking the smallest/largest eigenvalue of the matrix $[\Delta]$ and $||\xi||_2$ denotes the Euclidean norm of vector ξ. The time derivative of the function in Eq. (18) along the solutions of the system in Eq. (15) is:

$$\dot{V} = \xi^T P \dot{\xi} \tag{20}$$

$$\dot{V} = -\frac{1}{2} \mid \sigma \mid^{\frac{1}{2}} [\xi^T Q_0 \xi + d(-p_2\xi_1^2 + p_3\xi_1\xi_2) sign(\sigma)] \tag{21}$$

where

$$Q_0 = \begin{bmatrix} p_1\theta - p_2 K & -p_1 - p_2\theta + 2p_3 K \\ -p_1 - p_2\theta + 2p_3 K & p_2 \end{bmatrix} \tag{22}$$

Furthermore, taking into account the boundedness of model uncertainties, i.e.: $\mid \dot{d}(t) \mid \leq c$ (Proposition 1), we can find:

$$\dot{V} \leq -\frac{1}{2} \mid \sigma \mid^{\frac{1}{2}} \xi^T Q_1 \xi \tag{23}$$

with

$$Q_1 = \begin{bmatrix} p_1\theta - p_2 K - (p_3 - 2p_2)c & -p_1 - p_2\theta + 2p_3 K \\ -p_1 - p_2\theta + 2p_3 K & p_2 - p_3 c \end{bmatrix} \tag{24}$$

Now, setting $Q_1 = I_{2x2}$, and considering $P > 0$, we obtain the relations:

$$\begin{cases} p_2 > 1,\, p_3 = \dfrac{1+p_2}{c} and p_1 > \dfrac{cp_2^2}{p_2 - 1} \\ \theta = \dfrac{2p_3 + p_2 p_1 + 2p_3(p_3 - 2p_2)c}{2p_1 p_3 - p_2^2} \\ \quad k = \dfrac{p_1 + p_2\theta}{2p_3} \end{cases} \tag{25}$$

Using the fact that:

$$\mid \sigma \mid^{\frac{1}{2}} \leq ||\xi||_2 \leq \frac{V^{\frac{1}{2}}}{\lambda_{min}[P]} \tag{26}$$

We obtain:

$$\dot{V} \leq -\Phi V^{\frac{1}{2}} \tag{27}$$

With

$$\Phi = \frac{\lambda_{min}^{\frac{1}{2}}[P]\lambda_{min}[Q_1]}{\lambda_{max}[P]} \tag{28}$$

Finally, inequalities of Eqs. (19), (27) and (20) prove that the system Eqs. (10–12) is globally uniformly asymptotically stable [12]. □

4 Control design using adaptive super-twisting algorithm

The main advantages of an SMC are fast dynamic response and the robustness of closed loop system, which can be reached using a discontinuous function and high control gain. However, in order to design an SMC controller, it is necessary to know the uncertainty bounds, which is a difficult task from practical viewpoint. This is recompensed by increasing and overrating the controller gains. Nevertheless, high gains increase chattering [23]. In order to solve this issue, the authors in [24] have suggested an adaptive super-twisting algorithm (ASTA) where the controller gains are adapted dynamically to the parameter changes due to external disturbances and system uncertainties, which can reduce the chattering in steady-state. The main difference between the conventional super-twisting and adaptive super-twisting strategies is that in the latter, the control algorithm forces the sliding variable to a predefined neighborhood of the sliding surface. The overall ASTA strategy is depicted in Fig. 1. The gain K is adapted to the sliding mode output according to the following proposition:

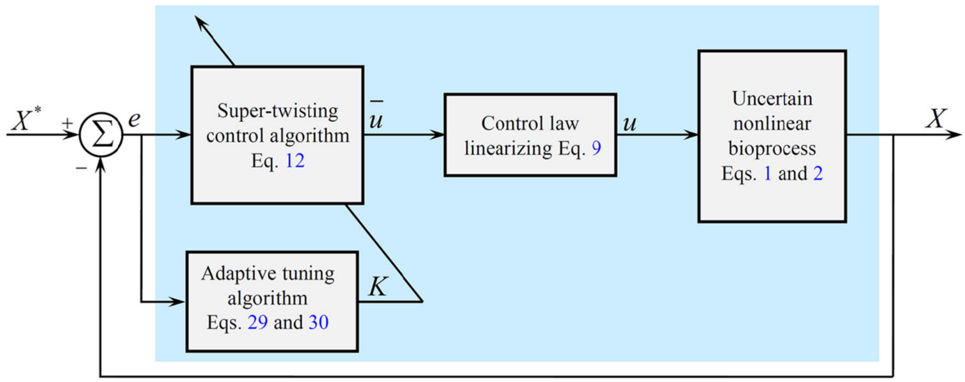


Fig. 1 Control structure of the ASTA strategy for a nonlinear bioprocess

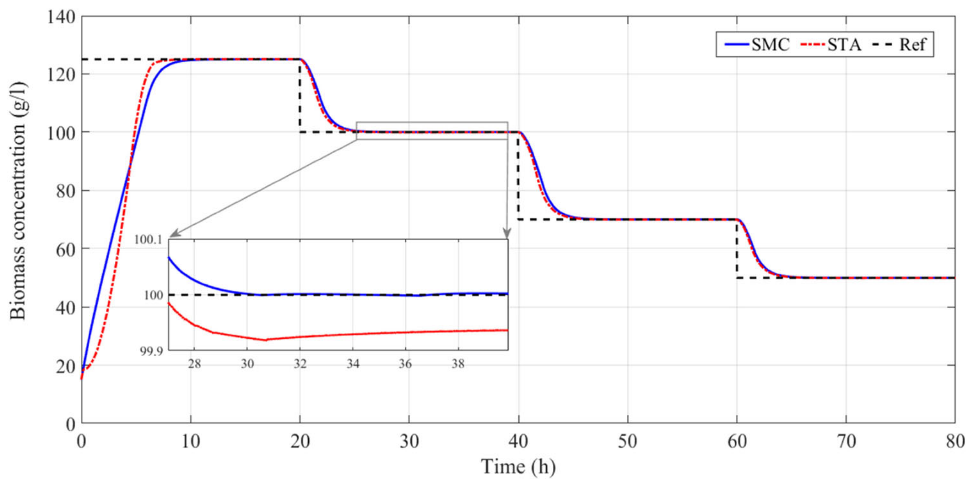


Fig. 2 Evolution of biomass concentration with SMC and STA

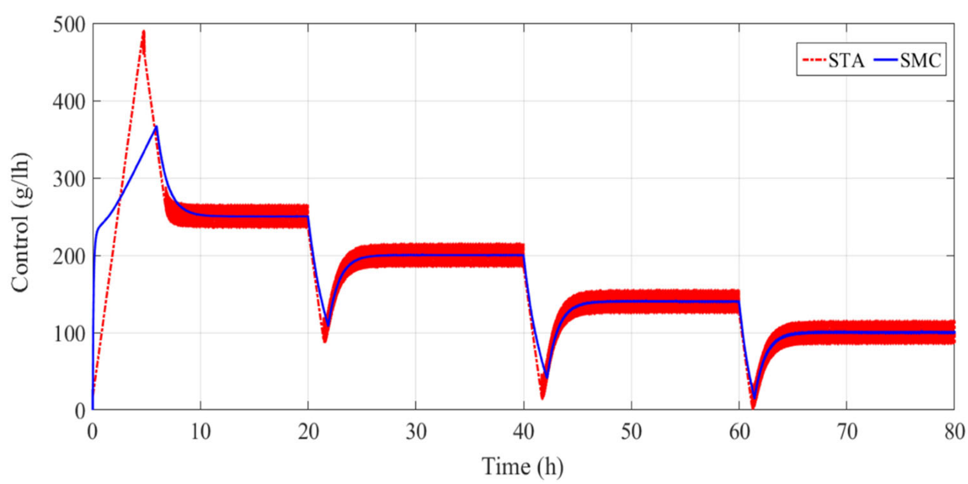


Fig. 3 The control signal for the SMC and STA

$$K(t) = \begin{cases} \int_0^t K_a \mid \sigma(x,t) \mid dt & if \mid \sigma(x,t) \mid > \epsilon > 0 \\ K_b \mid \eta \mid + K_c & if \mid \sigma(x,t) \mid \leq \epsilon \end{cases} \tag{29}$$

where

$$\tau \dot{\eta} + \eta = sign(\sigma(x,t)) \tag{30}$$

and K_a, K_c and τ are positive constants, K_b varies when there is a change in the surface sign at time t^*, where: t^* is the largest time value and by denoting t^{*-} the time just before t^*: $\sigma(x(t^{*-}, t^{*-}) > \epsilon$ and $\sigma(x(t^{*-}, t^{*-}) \leq \epsilon$ for a suitable positive parameter ϵ.

This adaptation law (Eqs. (29) and (33)) assurances the existence of a real sliding mode in finite time which preserves the main characteristics of the proposed STA in (Eqs. (12–16)).

Fig. 4 Zoom of the control signal for the SMC and STA

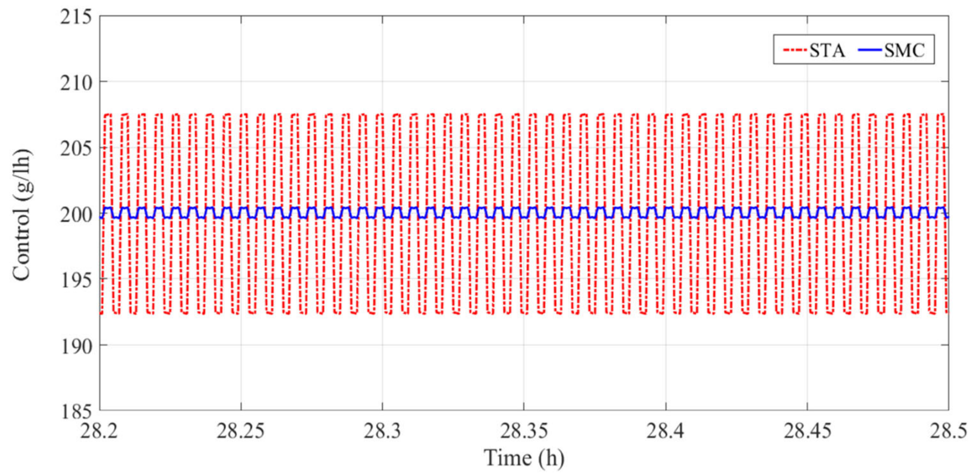


Fig. 5 Evolution of biomass concentration with SMC and STA in the uncertain case

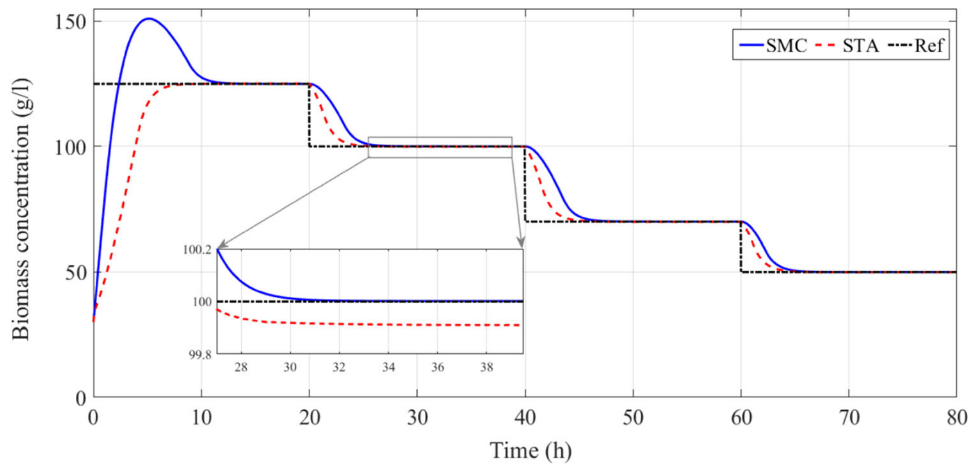
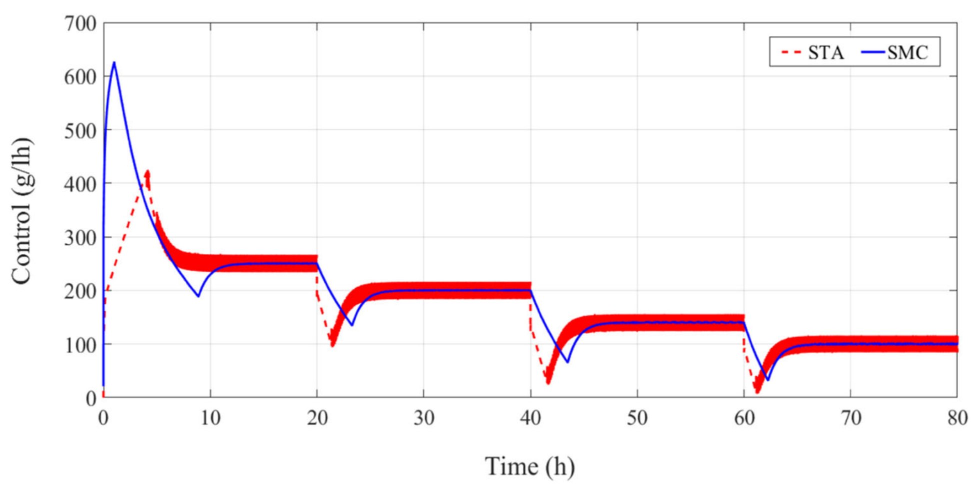


Fig. 6 The control signal for the SMC and STA in the uncertain case



5 Results and discussion

The performances of the STA and proposed ASTA, presented in Sects. 3 and 4 respectively, have been examined through numerical tests referring to the uncertain bioprocess model defined as follows:

$$\begin{cases} \dot{X} = \hat{\mu}(S)X - \hat{D}X \\ \dot{S} = -\hat{K}_1\hat{\mu}(S)X - \hat{D}S + u \\ e = X - X^* \end{cases} \tag{31}$$

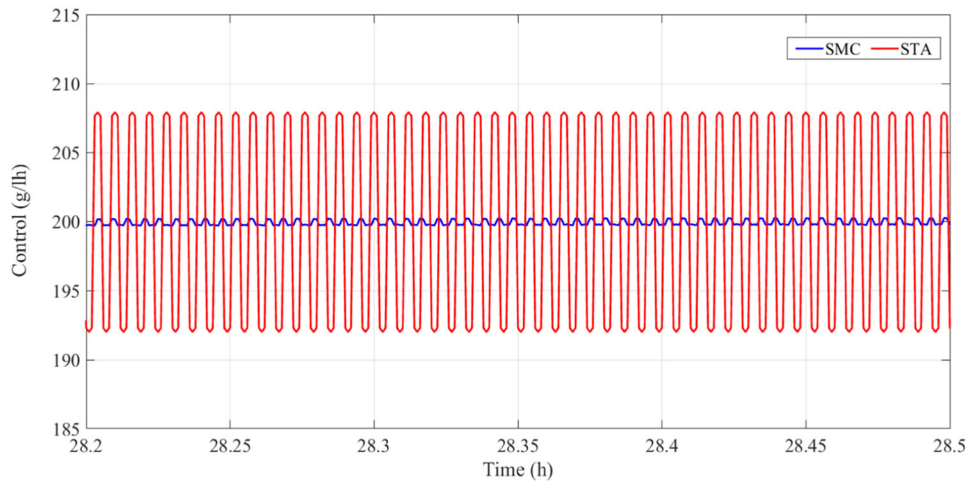


Fig. 7 Zoom of the control signal for the SMC and STA in the uncertain case

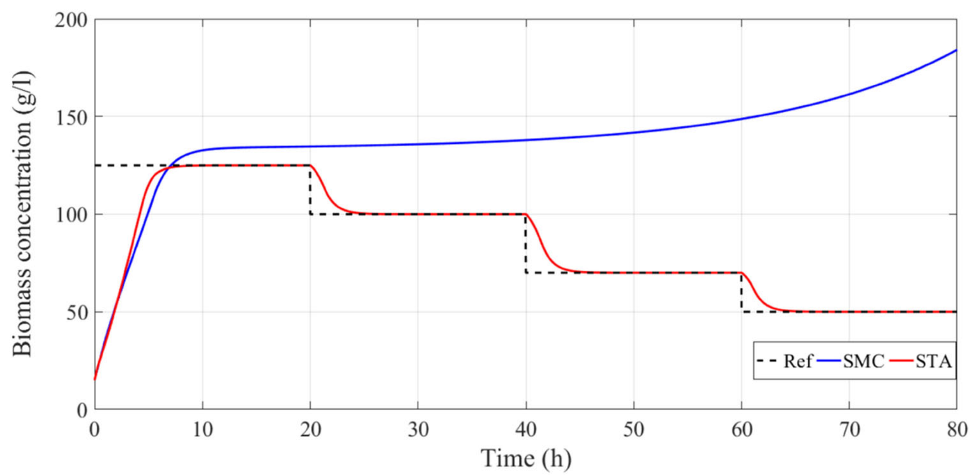


Fig. 8 Evolution of biomass concentration with SMC and STA in the uncertain perturbation case with large-perturbation

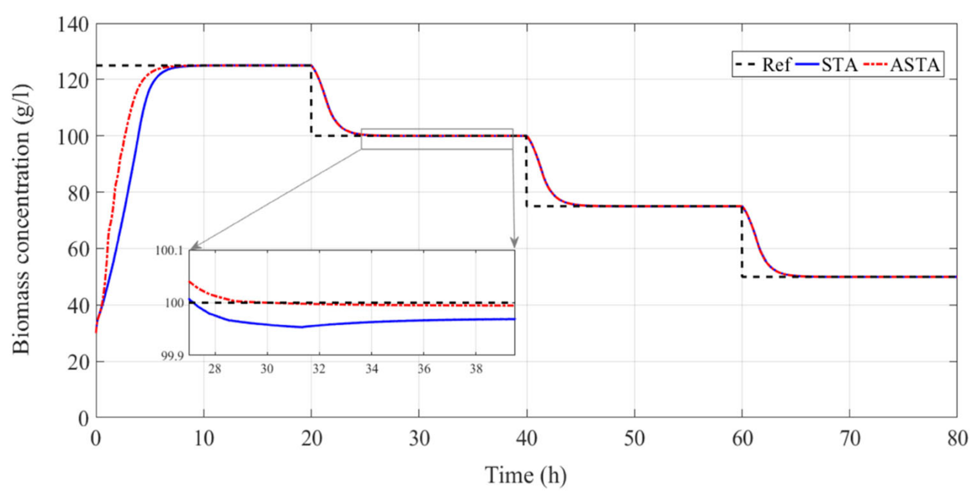


Fig. 9 Evolution of biomass concentration with STA and ASTA in the uncertain case

where

$$\hat{\mu}(S) = \mu + \delta\mu \tag{32}$$

$$\hat{D} = D + \delta D \tag{33}$$

with μ, D and K_1 are the nominal parameters of the system. $\hat{\mu}$ and $\hat{D}$ are the uncertain parameters, which might be obtained in practical cases from an offline identification method or estimated online through an adaptive method [30]. $\delta(\Delta)$ are unknown bounded parameters introduced here to model parametric uncertainties. The values of the nominal parameters have been chosen identical to those given in [30] as $\mu^* = 2.1h^{-1}$, $K_M = 10g/l$, $D = 0.2h^{-1}$ and $K_1 = 12$.

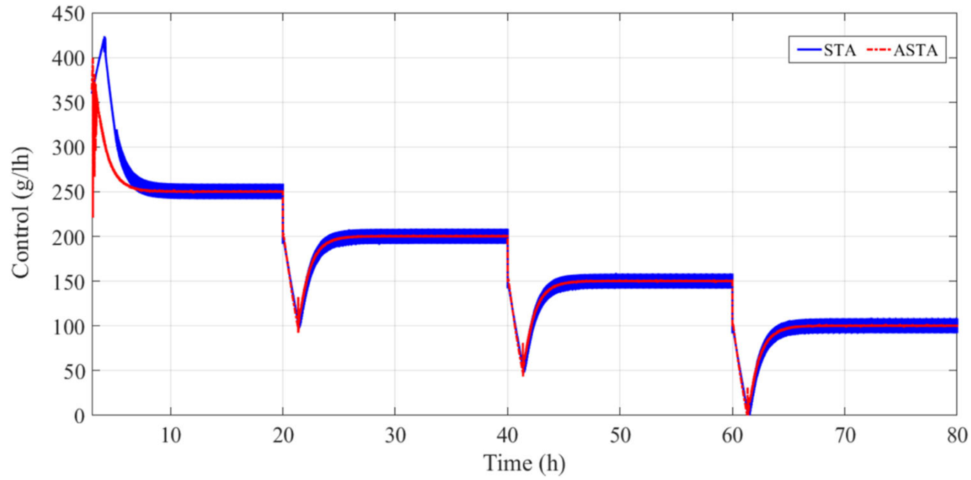


Fig. 10 The control signal for the STA and ASTA in the uncertain case

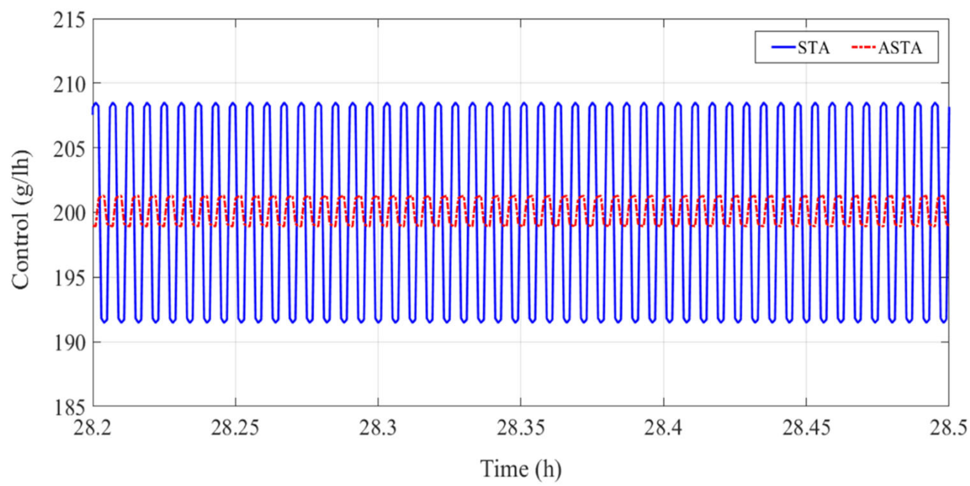


Fig. 11 Zoom of the control signal for the STA and ASTA in the uncertain case

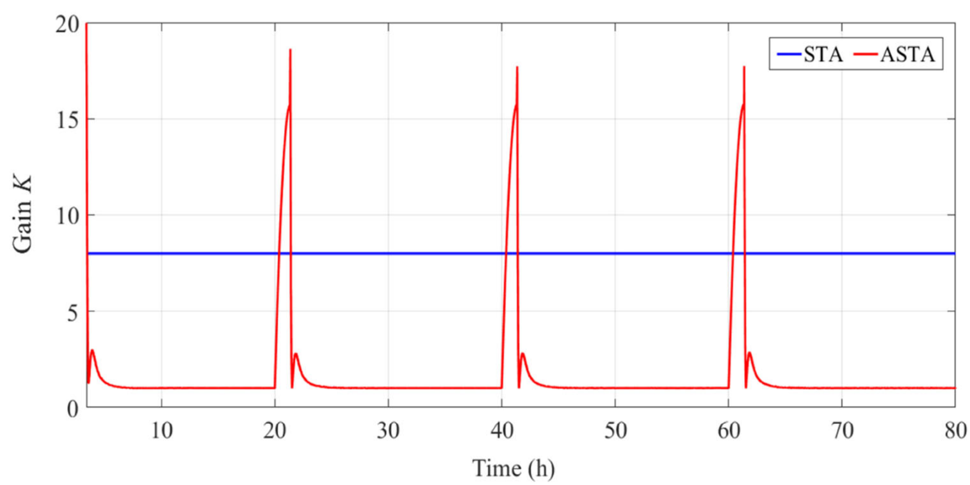


Fig. 12 Gain variation for the STA and ASTA

The simulated control task considers the problem of stabilizing the error e to zero in order to minimize the biomass concentration X to the pre-specified constant reference value X^* while preventing negative values in the control design. The main goal of simulation tests is to compare the performances of three controllers:

– The classical first-order sliding mode controller based on the equivalent control method with fixed gain ($K = 8$) defined as follows:

$$u(t) = \hat{K}_1\hat{\mu}(S)X - \frac{(\hat{\mu}(S) - \hat{D})^2}{\mu'(S)} - \frac{1}{\mu'(S)X}\hat{D})X + (\lambda(\hat{\mu}(S) - Ksign(\lambda(X - C) + (\hat{\mu}(S) - \hat{D})X)) \tag{34}$$

– The second-order sliding mode super-twisting controller based on fixed gain given by Eq. (12), where the control parameters are fixed in this case as $\theta = 101.03$ and $K = 8$.

– The adaptive super-twisting algorithm, which has been proposed in this paper, where the control parameters are chosen according to Eq. (29).

Figures. 2, 3 and 4 depict the system behavior under the classical SMC and the fixed gain super-twisting algorithms, when considering the model parameters are exactly known; i.e. $\delta\alpha = 0$, $\alpha = \{D, K, \mu\}$. These results show that when the system parameters are exactly known, the performances of the classical SMC technique are better than the STA in terms of steady-state error and chattering phenomena. The zoom in Fig. 2 highlights the high level of the regulation accuracy of the conventional SMC algorithm compared to the STA approach. One can note also the fast convergence of the conventional SMC algorithm. These results may be interpreted from the fact that, in contrast to the STA, the classical SMC algorithm contains a continuous part, the equivalent one, which accelerates the convergence of system states to the corresponding sliding manifold. Moreover, supporting the discontinuous control law with a continuous one allows to reduce the level of uncertainties. This later interprets clearly the difference between the control signals in Figs. 3 and 4.

The best knowledge of the system parameters is difficult in many practical cases. Thus, a second test was performed by considering parametric uncertainties.

Figures 5, 6, 7 and 8 show the performances of the above controllers under the consideration of bounded parameter uncertainties, i.e., $\delta\alpha \neq 0$, $\alpha = \{D, K, \mu\}$. As seen, the obtained results highlight the sensitivity of the classic sliding mode controller to the parameter uncertainties, due to the very long settling time, relative to the twisting controller response, that characterizes the response of such controller. The STA preserves the same performances as in the previous test, fast and relatively accurate response.

Figures 7 and 8 show the control inputs applied to the system using the SMC and super-twisting controllers. One can note the important chattering phenomenon that characterizes the STA which is a serious problem even in practical implementation of the SMC controller. In addition, the simulation results (Fig. 8) prove that large uncertainties lead to the instability of the system under consideration of the classical SMC, while it preserves the same performances under the STA, which proves the robustness of the super-twisting control algorithm.

The main conclusion of the tests is the insensitivity of the proposed STA to parametric uncertainties. However, the chattering phenomena remains the main drawback of the STA because of the large and the high frequencies of the control signals in Figs. 3, 4 and Figs. 7, 8.

As mentioned earlier, one approach to minimize the chattering phenomena is to introduce an SMC with an adaptive gain as proposed in Eqs. (29) and (30). A comparative study between the STA with fixed gain and the proposed Adaptive STA is reported in Figs. 9, 10 and 11. The time-evolution of the adaptive control gain is shown in Fig. 12. As shown in Fig. 9, the proposed adaptive control algorithm preserves the best performances in terms of settling time and regulation accuracy, which ensures the existence of a real sliding mode in the neighborhood of the chosen sliding manifold. Figures 10 and 11 show that the adaptation of the control gains allows a significant reduction of the chattering phenomena in the control input signal. Moreover, one can note that the proposed control algorithm drives the system to its desired state without the knowledge of neither the dynamic parameters of the system nor the bounds of uncertainties.

6 Conclusion

An adaptive super-twisting algorithm (ASTA)—a special algorithm of second-order sliding mode control techniques—has been developed in this paper for the control of a biotechnological process placed inside of a continuous stirred tank bioreactor (CSTB). Considering unknown structured and unstructured uncertainties, the optimal control parameters of the classical STA are dynamically adapted to ensure the finite time establishment of a real sliding mode. Numerical tests have been presented to illustrate the effectiveness of the proposed approach. Compared with classical STA and equivalent SMC approach, the proposed methodology ensures the best performances of the biotechnological bioprocess behavior in terms of the system stability, settling time of the system output, disturbances rejection and the softening of the control inputs.

For future work, one may exploit and combine the proposed adaptive algorithm with the basis of fractional order model based on fractional calculus concepts such as Riccati [31,32] and Bagley–Torvik equations [33] in order to achieve more effective tracking performance. It will also be a future task to develop a hardware and software platform for the practical implementation in real time of the proposed methodology.

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