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***Abstract*—This paper focuses on the analysis of the control problems in DC-DC buck switched mode power converter, and the study of an adaptive backstepping controller (ABSC), highlighting the properties and advantages of backstepping control and considering their exploitation in the stability analysis of such converter. The design of the proposed controller overcomes backstepping control problems by using an explicit dynamic model of the converter suitable for implementation. The effectiveness of the studied ABSC approach is tested when the converter is affected by various perturbations and the results are compared to those obtained by a backstepping controller (BSC), and a conventional PI controller (proportional - integral).**

Keywords— DC-DC Buck Converter; PWM; Backstepping control; Adaptive Backstepping control; Robustness

I. Introduction

The static DC-DC converters are generally used in power electronics systems for more and more important applications such as domestic applications (mobile phone, computers, household appliances, etc.), automotive industry, renewable energy systems (panels solar photovoltaic, wind turbines, etc.) and also in electrical energy transmission networks. Therefore, DC-DC converters have a very important role in energy conversion systems. Several circuit topologies are proposed for different applications. We can generally classify them by the basic topologies, which are the DC-DC converters of type Buck (lower), Boost (elevator) and Buck-Boost (inverter). The control method of static converters is associated with pulse width modulation ***(PWM)*** [1]. The ***PWM*** signal drives the converter to maintain a constant output voltage, while the duty cycle of the PWM signal is adjusted by some controllers. We can find in the literature many controllers for DC-DC converters that are designed based on linear control techniques [2]. This type of controller is known as a conventional ***PID*** controller. The design of ***PID's*** is based on a linear model of the power system operating at a given operating point. However, the static converters are highly complex due to their capacity of nonlinear and time varying elements. Thus, the parameters of the ***PID*** controller that are appropriate for one operating point may not be effective for another. This is the major inconvenience of the ***PID's*** design method, i.e. it does not guarantee the stability of the system under real operating conditions. Indeed, it is necessary to develop nonlinear controllers for which the mathematical model is inaccurate and cannot be easily obtained. Many nonlinear controllers have been introduced to improve the efficiency of DC-DC converters, including fuzzy logic [3], neural netework [4], sliding mode [5], model prediction [6] and backstepping control (***BSC***)[7]. Among those controllers, ***BSC*** is a nonlinear control approach that has been introduced for the control of variable structure systems (such as static converters), and the basic idea of the nonlinear backstepping controller is to synthesize a control law in a recursive way, that is to say step by step. However, the robustness of the backstepping control law is not guaranteed and the system dynamics may be insensitive to parameter variation and perturbations [8],[9].

In this context, we propose in this work a control approach using adaptive backstepping ***(ABCS)*** strategy to control a DC-DC buck converter. By reducing the time required to reach the equilibrium point, both convergence and the fast mitigation of perturbations are enhanced with the proposed approach. This paper is organized as follows: in the ***second section*** description of DC-DC buck converter is introduced. In the ***third section***, the backstepping control is detailed. ***Section fourth*** is devoted to the analysis of the simulation results of backsteppig controller, in the ***section fifth,*** the adaptive backstepping control is detailed with simulation and comparison with ***BSC*** and ***PI*** controllers and finally is the conclusion.

II. Buck Converter Modeling and Problem Statement

Fig. 1 shows the principal circuit diagram of the DC-DC buck power converter. It offers the possibility of providing a reduced (step-down) level of DC voltage at its load end [10]–[12]. Here, V_s represents the DC voltage across the load and i_L is the inductor current. The load of the converter is modeled as an effective resistance ***R***. The input DC voltage source is termed as V_{in}. A power electronic switch ***S*** is used for chopping the input supply and a power diode ***D*** is used for free-wheeling mechanism.

It is assumed that the converter is operating in continuous conduction mode (the current i_L crossing the inductance never get zero). There are two operating intervals of the converter, i.e. ***interval 1***, in which the switch ***S*** is turned ***On***, and ***interval 2***, in which the switch S is turned ***Off***.

- During the first interval, the diode ***D*** does not conduct and input source V_{in} acts as a sole source of energy to the buck converter.
- During the second interval, the diode ***D*** resumes electrical conduction ensuring flow of current through the inductor ***L***. Hence, the stored energy in the inductor drives the load resistance ***R*** during ***off*** mode of power switch ***S***.

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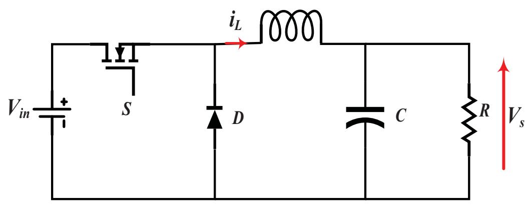

Fig. 1. Circuit of a buck DC-DC converter

Let $x_1 = i_L(t)$ and $x_2 = V_s(t)$ be the states of the DC-DC buck converter[13]. The dynamics model of buck power converter can be represented in state space as the following

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{L}x_2 + \mu\frac{V_{in}}{L} \\ \dot{x}_2 &= \frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \quad (1)$$

where $\mu = \{0,1\}$ is the control signal indicating the opening (***On*** mode) and closing (***Off*** mode) operation of switch ***S***.

The control objective is to obtain a faithful tracking of output voltage V_s, and also to ensure a satisfactory transient behavior.

III. Backstepping Controller

A. BSC designe

The backstepping design control is a recursive methodology. It involves a systematic construction of both the law of feedback control and Lyapunov's associated function [9],[14]. The backstepping controller will be designed in two steps since the DC-DC buck converter is a second order system.

- ***Step 1: Find, a virtual control law***

First of all, we define the error signal

$$e_1 = x_2 - V_{ref} \quad (2)$$

where V_{ref} is the reference voltage. By converging the e_1 to zero, we can get the desired result.

Using the system (1), the tracking of error derivative is written as follows

$$\dot{e}_1 = \dot{x}_2 - \dot{V}_{ref} \quad (3)$$

$$\dot{e}_1 = \frac{1}{C}x_1 - \frac{1}{RC}x_2 - \dot{V}_{ref} \quad (4)$$

The following Lyapunov function is considered

$$V_1 = \frac{1}{2}e_1^2 \quad (5)$$

In order to assure the asymptotic stability, the Lyapunov function V_1 must be positive definite and radially unbounded and its derivative with respect to time must be negative definite [15]. Taking the time derivative of equation (5), we get

$$\dot{V}_1 = e_1\dot{e}_1 \quad (6)$$

$$\dot{V}_1 = e_1\left(\frac{1}{C}x_1 - \frac{1}{RC}x_2 - \dot{V}_{ref}\right) \quad (7)$$

From this equation for the derivative of Lyapunov function to be negative, it is necessary to verify

$$\frac{1}{C}x_1 - \frac{1}{RC}x_2 - \dot{V}_{ref} = -K_1e_1 \quad (8)$$

From where

$$\frac{x_1}{C} = -K_1e_1 + \frac{1}{RC}x_2 + \dot{V}_{ref} \quad (9)$$

Using the values of $\frac{x_1}{C}$ from equation (9), equation (7) becomes

$$\dot{V}_1 = e_1\left(-K_1e_1 + \frac{1}{RC}x_2 + \dot{V}_{ref} - \frac{1}{RC}x_2 - \dot{V}_{ref}\right) \quad (10)$$

$$\dot{V}_1 = -K_1e_1^2 \quad (11)$$

Since the derivative of V_1 to be definitively negative, the value of K_1 must be defined positively, and equation (9) must be satisfied.

β is the stabilization function (virtual control), acts as a reference current for $\frac{x_1}{C}$. Then defined by

$$\beta = -K_1e_1 + \frac{1}{RC}x_2 + \dot{V}_{ref} \quad (12)$$

Hence the asymptotic stability of the system (1) in origin is reached.

- ***Step 2: Find μ, the original control input***

The second error variable, which represents the difference between the state variable $\frac{x_1}{C}$ and its desired value β, is defined by

$$e_2 = \frac{x_1}{C} - \beta \quad (13)$$

Or

$$\frac{x_1}{C} = e_2 + \beta \quad (14)$$

By differentiating equation (14), equation (4) becomes

$$\dot{e}_1 = e_2 + \beta - \frac{1}{RC}x_2 - \dot{V}_{ref} \qquad (15)$$

$$\dot{e}_1 = -K_1 e_1 + e_2 \qquad (16)$$

The derivative of e_2 can be defined as follows

$$\dot{e}_2 = \frac{\dot{x}_1}{C} - \dot{\beta} \qquad (17)$$

Therefore

$$\dot{e}_2 = \frac{\dot{x}_1}{C} + K_1\dot{e}_1 - \frac{1}{RC}\dot{x}_2 - \ddot{V}_{ref} \qquad (18)$$

$$\begin{aligned} \dot{e}_2 &= -K_1^2 e_1 + K_1 e_2 - \frac{1}{RC^2}x_1 \\ &+ x_2\left[\frac{1}{(RC)^2} - \frac{1}{LC}\right] + \mu\frac{V_{in}}{LC} - \ddot{V}_{ref} \end{aligned} \qquad (19)$$

To insure the asymptotic stability of the system (1) and the convergence of the errors e_1 and e_2 to zero, a composite Lyapunov function V_t is defined whose time derivative should be negative definite for all value of x_1 and x_2.

$$V_t = V_1 + \frac{1}{2}e_2^2 \qquad (20)$$

The derivative of V_t is:

$$\dot{V}_t = \dot{V}_1 + e_2\dot{e}_2 \qquad (21)$$

$$\dot{V}_t = -K_1 e_1^2 + e_2\left[e_1 + \dot{e}_2\right] \qquad (22)$$

For the derivative of V_t negative it is necessary to verify

$$e_1 + \dot{e}_2 = -K_2 e_2 \qquad (23)$$

$$\begin{aligned} \mu &= \frac{LC}{V_{in}}\left[e_1\left(K_1^2 - 1\right) - e_2\left(K_1 + K_2\right) + \frac{1}{RC^2}x_1\right] \\ &+ \frac{LC}{V_{in}}\left[\ddot{V}_{ref} - x_2\left(\frac{1}{(RC)^2} - \frac{1}{LC}\right)\right] \end{aligned} \qquad (24)$$

Proposition 1:

Consider the control system including the average ***PWM*** buck model (1) in closed-loop with the ***BSC*** controller (24), where the desired output voltage V_{ref} is sufficiently smooth and satisfies $0 < V_{ref} < V_{in}$ .Then, the equilibrium of the system (x_1, x_2, μ) = (I_d, V_{ref}, d) is asymptotically and locally stable where

$$I_d = \frac{V_{ref}}{R} \qquad \text{and} \qquad d = \frac{V_{ref}}{V_{in}}$$

B. BSC results

In this section, the numerical simulation of the DC-DC Buck converter has been developed and implemented in MATLAB/Simulink® environment. The backstepping controller presented in this section is applied to a buck converter. The parameters concerned and their values are indicated in Table I.

- ***Case 1:*** The converter is working under nominal condition, which include the reference voltage ***V_{ref} =12V*** with a nominal load ***R=6*** **Ω**. **Fig. 1** illustrates the simulation results.

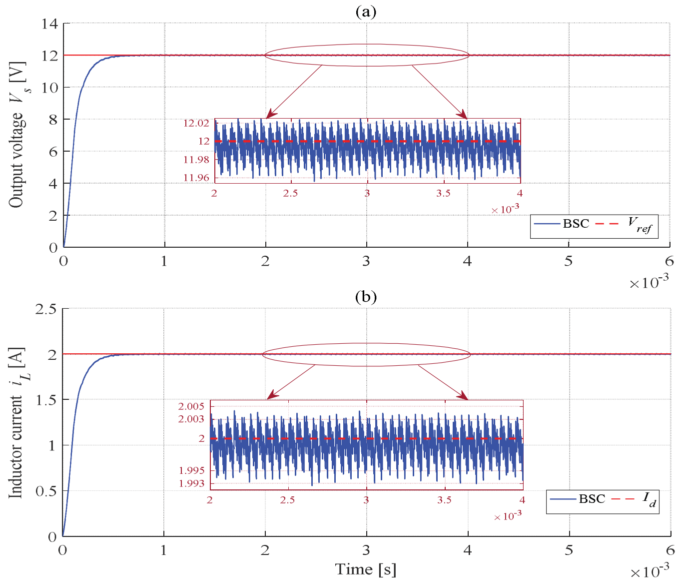

Fig. 2. Output voltage and current inductor of buck converter for 1st case.

- ***Case 2:*** In this case, a change of the reference voltage from $12\,V \rightarrow 9\,V$ then to is produced with a constant time step 0.02 sec. The simulation results are presented in **Fig. 3**.

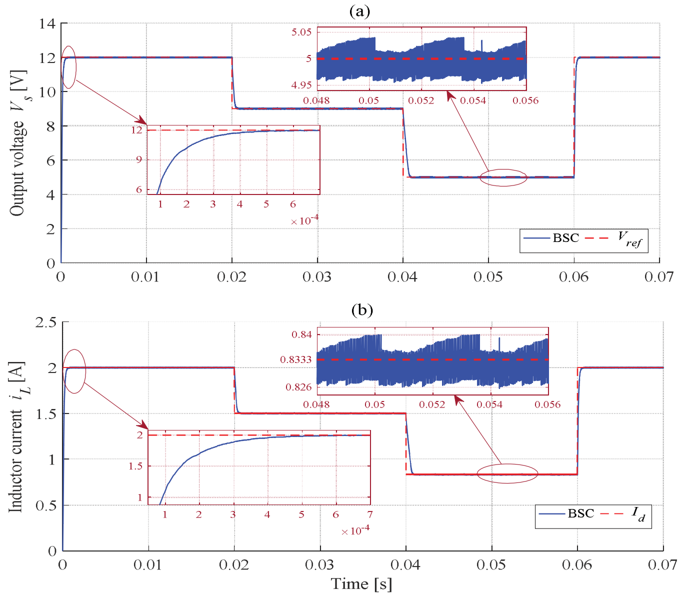

Fig. 3. Output voltage and current inductor of buck converter for 2nd case.

TABLE I. BUCK CONVERTER AND BACKSTEPPING PARAMETERS

	Parameters	*Values*
Buck converter	Input voltage Vin	24 V
	Capacitor C	202.5 μF
	Inductor L	98.58 μH
	Load R	6 Ω
	frequency Fs	20 kHz
Backstepping controller	Gain K_1	800
	Gain K_2	150

The ***BSC*** law shows the effectiveness of this approach to ensure the tracking of output voltage and load current to their reference values (***Case 1 and Case 2***). However, this technique can only be used for this system, whose parameters are perfectly known. This means in particular that the resistance of the load ***R*** is constant and invariable at all time. Thus, it presents the problem of asymptotic convergence. So, in real situations, the search for such a solution takes a lot of time and needs many simulations. In order to overcome these problems, we propose an adaptive backstepping controller, which will be detailed in the next section.

IV. ADAPTIVE BACKSTEPPING CONTROLLER DESIGN

A. ABSC designe

The goal of the control, as seen in the first section, is to force the output voltage of buck power converter x_2 to track the desired value $\boldsymbol{V_{ref}}$. The difference with respect to the previous section, lies in the fact that load resistance ***R*** is not known. To cope with such a model uncertainty the new controller (***ABSC***) will be given a learning capacity. More specifically, the controller to be designed should involve an online estimation of the unknown parameter $\theta = 1/R$. The obtained estimate is denoted $\hat{\theta}$. Just as in the non-adaptive case (***BSC***), the adaptive design procedure consists of two steps.

- ***First Step :***

By following closely ***step 1*** of the design of ***BSC***, we successively define the voltage error $e_1 = x_2 - V_{ref}$, the stabilization function

$$\beta = -K_1 e_1 + \frac{\hat{\theta}}{C} x_2 + \dot{V}_{ref} \tag{25}$$

and the error $e_2 = \frac{x_1}{C} - \beta$. In (25), $\boldsymbol{K_1}$ is an ***ABSC*** design parameter. With these definitions, one gets from model (1)

$$\dot{e}_1 = e_2 - K_1 e_1 - \frac{\tilde{\theta}}{C} x_2 \tag{26}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, signifies the error in estimating the parameters.

- ***Second Step:***

The derivative of $e_2 = \frac{x_1}{C} - \beta$ can be rewritten as follows:

$$\dot{e}_2 = \frac{\dot{x}_1}{C} - \dot{\beta} \tag{27}$$

From (26) we obtain

$$\dot{\beta} = -K_1 e_2 + K_1^2 e_1 + K_1 \frac{\tilde{\theta}}{C} x_2 + \frac{\dot{\hat{\theta}}}{C} x_2 + \frac{\hat{\theta}}{C^2}\left[x_1 - \theta . x_2\right] + \ddot{V}_{ref} \tag{28}$$

Which means with (27)

$$\dot{e}_2 = -\frac{1}{LC} x_2 + \mu \frac{V_{in}}{LC} + K_1 e_2 - K_1^2 e_1 - K_1 \frac{\tilde{\theta}}{C} x_2 - \frac{\dot{\hat{\theta}}}{C} x_2 - \frac{\hat{\theta}}{C^2}\left[x_1 - \theta . x_2\right] - \ddot{V}_{ref} \tag{29}$$

From (26)–(29), we will consider the following Lyapunov function:

$$V_t = \frac{1}{2} e_1^1 + \frac{1}{2} e_2^2 + \frac{1}{2\gamma} \tilde{\theta}^2 \tag{30}$$

where $\gamma > 0$ is a real constant and positive, called the parameter adaptation gain.

The derivative of $\boldsymbol{V_t}$ with respect to time is given by:

$$\dot{V}_t = e_1 \dot{e}_1 + e_2 \dot{e}_2 + \frac{1}{\gamma} \tilde{\theta} \dot{\tilde{\theta}} \tag{31}$$

$$\begin{aligned} \dot{V}_t = & -K_1 e_1^2 - K_2 e_2^2 + e_2 \Big[e_1 - \frac{1}{LC} x_2 + \mu \frac{V_{in}}{LC} - \ddot{V}_{ref} \\ & - K_1^2 e_1 + K_2 e_2 + K_1 e_2 - \frac{\hat{\theta}}{C^2}\left[x_1 - \hat{\theta} . x_2\right] - \frac{\dot{\hat{\theta}}}{C} x_2 \Big] \\ & + \tilde{\theta} . \left[e_2 \left(\frac{\hat{\theta}}{C^2} x_2 - K_1 \frac{1}{C} x_2 \right) - e_1 \frac{1}{C} x_2 - \frac{1}{\gamma} \dot{\hat{\theta}} \right] \end{aligned} \tag{32}$$

where $\boldsymbol{K_2}$ is an ***ABSC*** design parameter. The control and adaptation laws are respectively obtained by setting to zero the quantities between hooks in (32). Then one gets

$$\mu = \frac{LC}{V_{in}} \left[e_1 \left(K_1^2 - 1 \right) - e_2 \left(K_1 + K_2 \right) + \frac{1}{LC} x_2 + \ddot{V}_{ref} \right] + \frac{LC}{V_{in}} \left[\frac{\dot{\hat{\theta}}}{C} x_2 + \frac{\hat{\theta}}{C^2} \left[x_1 - \hat{\theta} . x_2 \right] \right] \tag{33}$$

with

$$\dot{\hat{\theta}} = \gamma.\frac{x_2}{C}\left[e_2\left(\frac{\hat{\theta}}{C} - K_1\right) - e_1\right] \qquad (34)$$

Proposition 2.

Consider the control system including the average ***PWM*** buck DC-DC power converter model (1) in closed-loop with the ***ABSC*** controller (33), where the desired output voltage V_{ref} is sufficiently smooth and satisfies $0 < V_{ref} < V_{in}$. Then, the equilibrium of the system, $(x_1, x_2, \mu) = (\hat{I}_d, V_{ref}, d)$ is locally and asymptotically stable where

$$\hat{I}_d = \hat{\theta}.V_{ref} \qquad and \qquad d = \frac{V_{ref}}{V_{in}}$$

B. ABSC results

The components of the controlled buck power converter have the same values as in Section 4^{th}. The ***ABSC*** controller design parameters have the following values

$$K_1 = 800;\ K_2 = 150;\ \gamma = 9^{-10}.$$

- ***Scenario 1:*** The converter is working with the reference voltage $V_{ref} = 12\,V$ and a load $R = 10\Omega$. The results are compared with those obtained with ***BSC*** and the ***PI*** controllers under the same operating conditions. The simulation results are illustrated by ***Fig. 4***.

In this scenario, it can be seen through the simulation results that the two controllers (***ABSC*** and ***PI***) provide the desired responses, but the response time is very short with the ***ABSC*** controller by contributing to the response time with the ***PI*** controller.

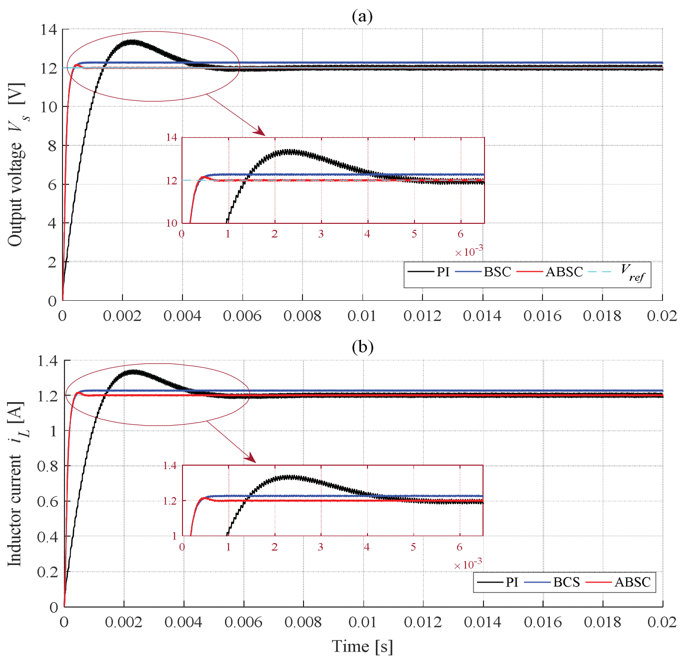

Fig. 4. Output voltage and current inductor of buck converter for 1st scenario.

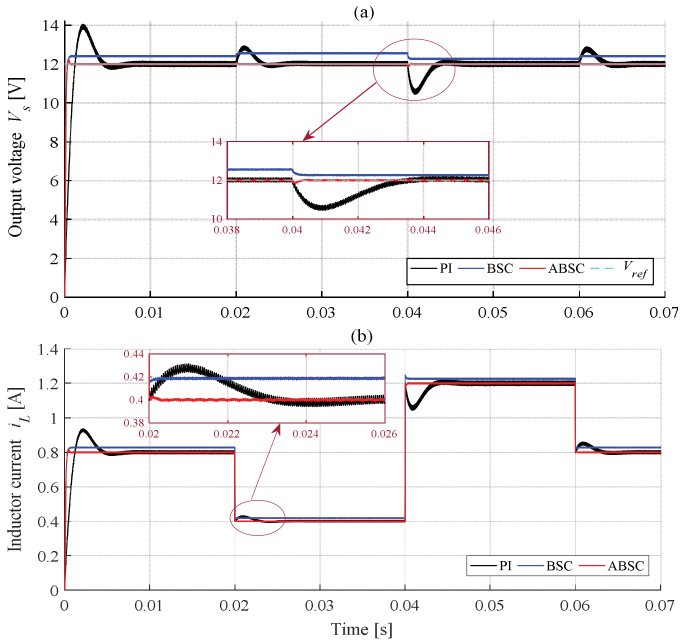

Fig. 5. Output voltage and current inductor of buck converter for 2nd scenario.

- ***Scenario 2:*** Uncertainties are taken into consideration. From the operating point of the first scenario, the ***ABSC*** controller performance is analyzed in the presence of load variations. This latter is varied between values $R = 15\,\Omega$ to $R = 30\,\Omega$ then to $R = 10\,\Omega$ during a periodic time interval. The responses of the system are given in ***Fig. 5.***

According to the simulation results, the inductor current has changed the operating point as shown in ***Fig. 5(b)***. This change causes oscillations in the system responses with the ***PI*** controller. The proposed controller (***ABSC***) is more efficient than the ***PI*** controller, in that it damped and canceled the oscillations in the responses of the considered system with a fast response time.

- ***Scenario 3:*** In this scenario, a change of the reference voltage of $12\,V$ *to* $9\,V$ then to $5V$ is produced with a constant time step $t = 0.02.sec$. The simulation results are shown in ***Fig. 6.***

It can be seen from the results of simulations during the variation of the reference voltage that the responses of the converter to a large oscillation especially at the current level with the conventional controller ***PI***, and the ***ABSC*** controller show an improvement in the suppression of the oscillations in short time.

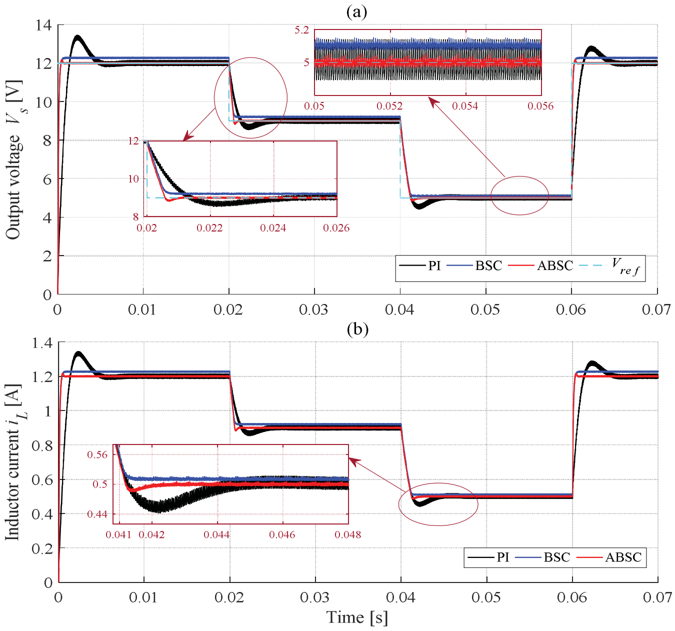

Fig. 6. Output voltage and current inductor of buck converter for 3rd scenario.

- ***Scenario 4: Fig.* 7** show the efficiency of the ***ABSC*** when the perturbation considered is represented by a periodic change of the input voltage. This latter has been changed from 36 V to 24 V at the moment t = 0.02sec and from 24V to 48V at the moment 0.04 sec, while the reference voltage is fixed at the value $V_{ref} = 12V$.

As shown in ***Fig.* 7,** changing the input voltage returns the large oscillations converter with the ***PI*** controller. The proposed ***ABSC*** controller provides system stability with a fast response time.

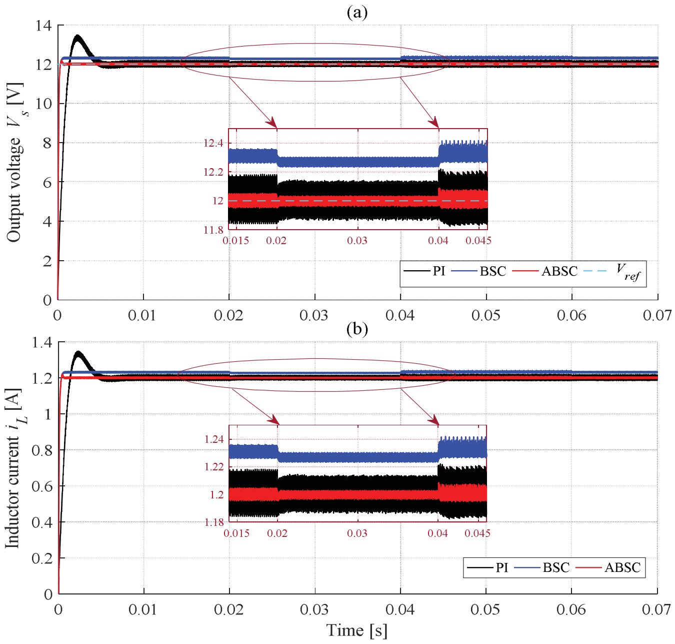

Fig. 7. Output voltage and current inductor of buck converter for 4th scenario.

V. Conclusion

In this work, a robust and nonlinear adaptive backstepping controller applied to a DC-DC buck converter has been presented. The asymptotic stability of the controlled system is verified via Lyapunov stability analysis. The backstepping strategy proves its robust feature to sufficient control and stabilize a nonlinear system, however, it suffers from the problem of asymptotic convergence which is the major inconvenient of this approach. To overcome this problem, an adaptive backstepping controller has been proposed. The obtained results lead us to conclude that the integration of the proposed adaptive mechanism in this work compensates for all the perturbations. It was also found that the main objectives set and planned have been reached.

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