Takagi-Sugino fuzzy model of the Anaerobic Digestion of Organic Wastes

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Abstract—**the major problem of biotechnological process supervision is the complexity and the non-linearity of mathematical model. However, the non-linear model has been transformed into a linear one using multimodal. In this paper, an application of Takagi-Sugeno modeling method of Anaerobic Digestion (AD) of organic wastes was studied and analyzed. Results obtained of the multimodal will be verified by computer simulations under Matlab/Simulink environment. The referred biotechnological process is described by a second-order non-linear model based one-stage reaction scheme.**

Keywords—

***Anaerobic digestion, Organic Wastes, Mathematical model, multimodal, Takagi-Sugino fuzzy model*.**

I. Introduction

Anaerobic digestion (AD) is a biotechnological process widely used in life processes and a promising method for solving some energy and ecological problems in agriculture and agro industry.

In such kind of processes, generally carried out in continuously stirred tank bioreactors, the organic matter is depolluted by microorganisms into biogas (methane and carbon dioxide) and compost in the absence of oxygen [1]. The biogas is an additional energy source which can replace fossil fuel sources. It therefore has a direct positive effect on greenhouse gas reduction. Unfortunately this process is very complex, may sometimes become very unstable and thus needs more investigations.

An active research problem is to better understand the dynamics of growth and death of the different populations of the complex community of bacteria acting during Anaerobic Digestion (AD) processes.

Many mathematical models of this process are known [2-3]. Generally they are very complex nonlinear sets of ordinary differential equations with a great number of unknown coefficients, which make them hard applicable for control purposes. More over, one way to cope with such difficulty is to develop a nonlinear model composing of a number of sub-models which are simple, understandable, and responsible for respective sub-domains. The idea of multi-model approach [1,4] is not new, but the idea of fuzzy modeling [5] using the concept of the fuzzy sets theory offers a new technique to build multi-models of the process based on the input-output data or the original mathematical model of the system.

The fuzzy model proposed by Takagi and Sugeno [5] is described by fuzzy IF-THEN rules which represent local input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models. In this article, the nonlinear dynamical system can be represented by Takagi-Sugeno fuzzy models to high degree of precision. In fact, it is proved that Takagi-Sugeno fuzzy models are universal approximations of any smooth nonlinear system [6,7] .

This paper is organized as follows. firstly, a description of the AD is presented in section II. In section III, the mathematical model is given. Section IV the TS approach is presented. In section VI computer simulation results are shown and discussed .Finally the reported work is concluded.

II. Mathematical Modeling of the AD Process

Anaerobic digestion is a multistep process involving the action of multiple microbes. Usually, such processes contain a particular step, the so-called rate limiting or rate-determining step, which, being the slowest, limits the rate of the overall process. Lawrence [8] defined as limiting step "that step which will cause process failure to occur under imposed conditions of kinetic stress". The first attempts for modeling anaerobic digestion led to models describing only the limiting step. However, during a wide range of operating conditions, the limiting step is not always the same. It may depend on wastewater characteristics, hydraulic loading, temperature, etc. Andrews [8] for example considered acetogenic methanogenesis as the limiting, O'Rourke [8] – the conversion of fatty acids to biogas, and Eastman and Ferguson [8] – the hydrolysis of biodegradable suspended solids.

It is apparent that the "limiting step hypothesis" leads to simple and readily usable models.

Such models, however, do not describe very well the anaerobic bioreactor (digester) behavior, especially under transient operating conditions [8].

The Graef and Andrews model [9] involves only the acetoclastic methanogens. The conversion of volatile fatty acids into biogas is considered limiting. This is the first and the most simplified model for description of the AD process describing the process rate limiting methanogenic step [2, 10] represented by the following differential equations:

$$\frac{dX}{dt} = \mu X - DX \quad (1)$$

$$\frac{dS}{dt} = -K_1 \mu X + D(S_{in} - S) \quad (2)$$

$$Q = K_2 \mu X \quad (3)$$

where S – substrate (acetate) concentration, $g \cdot dm^{-3}$; X – biomass concentration, $g \cdot dm^{-3}$; D – dilution rate, day^{-1}; S_{in} – concentration of inlet organics, $g \cdot dm^{-3}$; Q – biogas flow rate, $dm^3 \cdot day^{-1}$; μ – specific growth rate, day^{-1}; K_1 and K_2 are yield coefficients. The specific growth rate (μ) is with one from the following expressions:

– Monod type:

$$\mu = \frac{\mu_{max} S}{k_s + S} \quad (4)$$

– Contois type:

$$\mu = \frac{\mu_{max} S}{k_m X + S} \quad (5)$$

– Haldane type:

$$\mu = \frac{\mu_0 S}{k_s + S + S^2/k_i} \quad (6)$$

Where $\mu_{max}, k_s, k_m, \mu_0$, and k_i are kinetic coefficients.

According to this model, a digester is expected to fail whenever, for some reason, the fatty acid concentration is increased. This causes a drop in the pH and a rise in the acetic acid concentration. This in turn causes a drop in the growth rate of the methanogenic population, until they are washed out, if the situation is prolonged. It has been proved that this model poses single maximum of the static characteristic $Q = Q(D)$ [2]. This simple nonlinear model is very useful for testing different control algorithms.

III. INTRODUCTION TO FUZZY TAKAGI-SUGENO MODEL

The T-S models represent a very interesting mathematical formulation of nonlinear systems. These can therefore be easily represented regardless of their complexity with a simple structure based on a nonlinear combination of a set of linear models [11], [12], [13]. This simple structure with interesting properties, make them easily exploitable from a mathematical point of view.

A. Representation of Fuzzy Takagi-Sugeno Model:

Fuzzy models of Takagi-Sugeno are represented by fuzzy rules like "IF-THEN" [14]. The i^{th} fuzzy rule of continuous TS model is then written as:

$$\text{if } F_1(t) \text{ is } F_1^i(Z_1(t)) \text{ and } \dots \text{ and } F_p(t) \text{ is } F_p^i(Z_p(t))$$

$$\text{then } \begin{cases} x(t+1) = A_i x(t) + B_i u \\ y(t) = C_i x(t) + D_i u \end{cases}, \ i=1,2,\dots,r$$

where $F_j^i(Z_j(t))$ for *j=1,2,…,r* are fuzzy sets, r the number of fuzzy rules, $Z_j(t)$ premises variables that depend on the input and/or state of the system, $x(t) \in R^n, y \in R^q, u \in R^m$ respectively represent the state vector, the output vector and the control vector. $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{q\times n}$, $D_i \in R^{q\times m}$ are matrices describing the system dynamics.

Each rule is assigned a weight noted $\mu_i(z(t))$ which depends on the degree of membership of the premise variables $Z_j(t)$ in the fuzzy subsets $F_j^i(Z_j(t))$ and the connector "and" connecting the premises selected such that:

$$\mu_i(z(t)) = \prod_{i=1}^{p} F_j^i\left(Z_j(t)\right) \ , i = 1,2,\dots,r \quad (7)$$

$F_j^i(Z_j(t))$ represent the value of the membership function $Z_j(t)$ to the fuzzy set F_j^i we then have the following properties:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(z(t)) > 0 \\ \mu_i(z(t)) \geq 0 \ , \ i = 1,2,\dots,r \end{cases} \quad (8)$$

Finally, the defuzzification of the fuzzy model provides the state representation of a nonlinear model by interconnecting local time invariant models by non-linear activation functions. We obtain:

$$\begin{cases} x(t+1) = \frac{\sum_{i=1}^{r} \mu_i(z(t))\{A_i x(t) + B_i u\}}{\sum_{i=1}^{r} \mu_i(z(t))} \\ y(t) = \frac{\sum_{i=1}^{r} \mu_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} \mu_i(z(t))} C_i x(t) \end{cases} \quad (9)$$

B. Construction of Fuzzy Takagi-Sugeno Model:

To obtain a TS model include three approaches are widely used in the literature. The first approach relies on the identification techniques [15]. The second approach is based on the linearization of the nonlinear model around several operating points. The third approach is based directly on the analytical knowledge of the nonlinear model. It is known as the nonlinear transformation sectors [16], [17], [18]. The principle thereof is based on a polytopic convex nonlinear transformation under a dynamic system.

Unlike the first two approaches that give an approximation of the nonlinear model, this third method provides a representative T-S accurately the non-linear model. Note that the nonlinear sector approach allows to associate an infinity of T-S models for nonlinear system based on the division of nonlinearities achieved [17]. A systematic approach to cutting nonlinear areas based on the following lemma:

***Lema1.1:Transformation Convex Polytopic (TPC)* [16]**

Let $z(x(t), \mu(t))$ a bounded continuous function on the domain $D \subset R^n \times R^m$ values in R, with $x(t) \in R^n$, $u(t) \in R^m$

Then there exist two functions $(i = 1, 2)$

$$F_i : D \rightarrow [0, 1]$$

$$(x(t), u(t)) \rightarrow F_i(x(t), u(t))$$

With $F_1(x(t), u(t)) + F_2(x(t), u(t)) = 1$ *such as*

$$z(x(t), u(t)) = F_1(x(t), u(t)) z_1 + F_2(x(t), u(t)) z_2$$

for all $z_1 \geq \max_{x,u \in D}\{z(x, u)\}$ and $z_2 \leq \min_{x,u \in D}\{z(x, u)\}$

The functions F_1 and F_2 are defined by:

$$F_1(x(t),u(t)) = \frac{z(x(t),u(t)) - z_2}{z_1 - z_2}$$

$$F_2(x(t),u(t)) = \frac{z_1 - z(x(t),u(t))}{z_1 - z_2}$$

Where $z_1 = max_{x,u}\{z(x,u)\}$, $z_2 = min_{x,u}\{z(x,u)\}$

C. Form Quasi-Linear Parameter Variables "Quasi-LPV":

The first step is to convert the nonlinear model into a"quasi-linear variable parameters" called "quasi-LPV" model. It is given by:

$$\begin{cases} x(t+1) = A(x,u)x + B(x,u)u \\ y(t) = C(x,u)x + D(x,u)u \end{cases}$$

There are several possible choices for the quasi LPV form this depends on the choice of premise variables, whose affects the number of sub-models and the overall structure of the model, and on the feasibility of the system. We must also ensure observability/controllability of the overall system, and for each sub-model [19]

IV. APPLICATION OF FUZZY TAKAGI-SUGENO MODEL

The anaerobic digestion process of organic waste represented by the system of differential equations (Eq1,2,3) has three terms nonlinear.

The goal is to derive a T-S fuzzy model from the above given nonlinear system equations by the sector nonlinearity approach as if the response of the T-S fuzzy model in the specified domain exactly matches with the response of the original system with the same input.

The following steps should be taken to derive the T-S fuzzy model of (Eq.1,2,3). For simplicity, we assume that (μ, X, S)mimited. Here μ, Xand S are nonlinear terms in the last equations so we make them as our fuzzy variables.

Generally they are denoted as z_1, z_2 and z_3 are known as premise variables that may be functions of state variables, input variables, external disturbances and/or time. Therefore $z_1 = \mu$, $z_2 = X$ and $z_3 = S$. Equation (Eq.1,2,3) can be written as:

$$\dot{x} = Ax + Bu$$

Where $x = \begin{bmatrix} X \\ S \end{bmatrix}$ is state victor, $u = D$, $A = \begin{bmatrix} \mu & 0 \\ -k_1\mu & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -x \\ (S_{in}-S) \end{bmatrix}$

The matrix A and B are non-linear and non-stationer matrix. Then the system will be given by:

$$\dot{x} = \begin{bmatrix} \boldsymbol{\mu} & 0 \\ -k_1\boldsymbol{\mu} & 0 \end{bmatrix} x + \begin{bmatrix} -\boldsymbol{x} \\ (S_{in} - \mathbf{S}) \end{bmatrix} u$$

The first step for any kind of fuzzy modeling is to determine the fuzzy variables and fuzzy sets or so-called membership functions. Although there is no general procedure for this step and it can be done by various methods predominantly trial and error, in exact fuzzy modeling using sector nonlinearity, it is quite routine. It is assumed in this article that the premise variables are just functions of the state variables for the sake of simplicity. This assumption is needed to avoid a complicated defuzzification process of the fuzzy controllers [20].

To acquire membership functions, we should calculate the minimum and maximum values of μ, X and S, they are obtained from[21]:

Table I gives typical values of different variables and parameters of the bioprocess.

TABLE I. different bioprocess variables and parameters values

Symbol	Signification	Unit	Typical value
X	Concentration of bacteria	$g.l^{-1}$	0.025 – 0.500
S	Concentration of soluble organics	$g.l^{-1}$	0.025 – 0.125
μ	Specific growth rates of bacteria in the reactor	day^{-1}	0.020 – 0.140
D	Dilution rate	day^{-1}	0.025 – 0.125
$\mathbf{S_{in}}$	Influent concentration of organic matter	$g.l^{-1}$	0.300 – 3.000
Q	Biogas flow rate	$l\,gas.(l\,liquid^{-1}\,day^{-1})$	0.020 – 0.700
$\mathbf{K_1}$	Yield coefficient	–	6.70
$\mathbf{K_2}$	Yield coefficient	$l\,gas.g^{-1}$	16.78

$maxz_1 = 0.020$, $minz_1 = 0.140$
$maxz_2 = 0.025$, $minz_2 = 0.500$
$maxz_3 = 0.025$, $minz_3 = 0.125$

Therefore μ , X and S can be represented by for membership functions M_1, M_2, N_1, N_2, P_1 and P_2as follows:

$$z_1(t) = \mu(t) = M_1(z_1) \times 0.020 + M_2(z_1) \times 0.140$$
$$z_2(t) = X(t) = N_1(z_2) \times 0.025 + N_2(z_2) \times 0.500$$
$$z_3(t) = S(t) = P_1(z_3) \times 0.025 + P_2(z_3) \times 0.125$$

And because M_1, M_2, N_1, N_2, P_1 and P_2 are actually fuzzy sets according to fuzzy mathematics,

$$M_1(z_1) + M_2(z_1) = 1,$$
$$N_1(z_2) + N_2(z_2) = 1,$$
$$P_1(z_3) + P_2(z_3) = 1.$$

Here, we can generalize that the i^{th} rule of the continuous T-S fuzzy models are of the following forms:

Model Rule i:

$$IF\ z_1(t) is\ M_{i1} and \ldots and\ z_p(t) is\ M_{iP}$$

$$THEN \quad \begin{cases} x(t+1) = A_i x(t) + B_i u \\ y(t) = C_i x(t) + D_i u \end{cases} \quad i = 1,2,\ldots,r$$

Here, M_{ij} is the fuzzy set and r is the number of model rules; x(t)is the state vector,u(t) is the input vector, y(t) is the output vector, $A_i$is the square matrix with real elements and $z_1(t),\ldots,z_1(t)$ are known premise variables as mentioned before. Each linear consequent equation represented by $A_i x(t) + B_i u$ is called a subsystem.

The number of sub-model is given by $2^r$with r the number of variable premise, we have $2^3 = 8$ sub-models, which are obtained from possible combination of term's (non-constants) limits. The three non linear term's variation are summarized in **TABLE II**.

TABLE II. *non-linear term's combination:*

μ	X	S
$Minz_1$	$Minz_2$	$Minz_3$
$Minz_1$	$Minz_2$	$Maxz_3$
$Minz_1$	$Maxz_2$	$Minz_3$
$Minz_1$	$Maxz_2$	$Maxz_3$
$Maxz_1$	$Minz_2$	$Minz_3$
$Maxz_1$	$Minz_2$	$Maxz_3$
$Maxz_1$	$Maxz_2$	$Minz_3$
$Maxz_1$	$Maxz_2$	$Maxz_3$

Where the subsystems are determined as:

$$\begin{cases} A_1 = \begin{bmatrix} 0.020 & 0 \\ -0.02k_1 & 0 \end{bmatrix} \\ B_1 = \begin{bmatrix} -0.025 \\ (S_{in}-0.025) \end{bmatrix} \end{cases}, \begin{cases} A_2 = \begin{bmatrix} 0.020 & 0 \\ -0.02k_1 & 0 \end{bmatrix} \\ B_2 = \begin{bmatrix} -0.025 \\ (S_{in}-0.125) \end{bmatrix} \end{cases}$$

$$\begin{cases} A_3 = \begin{bmatrix} 0.020 & 0 \\ -0.02k_1 & 0 \end{bmatrix} \\ B_3 = \begin{bmatrix} -0.500 \\ (S_{in}-0.025) \end{bmatrix} \end{cases}, \begin{cases} A_4 = \begin{bmatrix} 0.02 & 0 \\ -0.02k_1 & 0 \end{bmatrix} \\ B_4 = \begin{bmatrix} -0.500 \\ (S_{in}-0.125) \end{bmatrix} \end{cases}$$

$$\begin{cases} A_5 = \begin{bmatrix} 0.140 & 0 \\ -0.140k_1 & 0 \end{bmatrix} \\ B_5 = \begin{bmatrix} -0.025 \\ (S_{in}-0.025) \end{bmatrix} \end{cases}, \begin{cases} A_6 = \begin{bmatrix} 0.140 & 0 \\ -0.140k_1 & 0 \end{bmatrix} \\ B_6 = \begin{bmatrix} -0.025 \\ (S_{in}-0.125) \end{bmatrix} \end{cases}$$

$$\begin{cases} A_7 = \begin{bmatrix} 0.140 & 0 \\ -0.140k_1 & 0 \end{bmatrix} \\ B_7 = \begin{bmatrix} -0.500 \\ (S_{in}-0.025) \end{bmatrix} \end{cases}, \begin{cases} A_8 = \begin{bmatrix} 0.140 & 0 \\ -0.140k_1 & 0 \end{bmatrix} \\ B_8 = \begin{bmatrix} -0.500 \\ (S_{in}-0.125) \end{bmatrix} \end{cases}$$

Where: $S_{in} = 2.00$ and $k_1 = 6.70$

Now, $\dot{x}$ can be derived out of defuzzification process as:

$$\dot{x}(t) = h_1(z(t))A_1x(t) + h_2(z(t))A_2x(t) + h_3(z(t))A_3x(t) + h_4(z(t))A_4x(t) + h_5(z(t))A_5x(t) + h_6(z(t))A_6x(t) + h_7(z(t))A_7x(t) + h_8(z(t))A_8x(t) \quad (10)$$

Where: $h_1(z(t)) = M_1(z_1(t)) \times N_1(z_2(t)) \times P_1(z_3(t))$

$$h_1(z(t)) = M_1(z_1(t)) \times N_1(z_2(t)) \times P_2(z_3(t))$$
$$h_1(z(t)) = M_1(z_1(t)) \times N_2(z_2(t)) \times P_1(z_3(t))$$
$$h_1(z(t)) = M_1(z_1(t)) \times N_2(z_2(t)) \times P_2(z_3(t))$$
$$h_1(z(t)) = M_2(z_1(t)) \times N_1(z_2(t)) \times P_1(z_3(t))$$
$$h_1(z(t)) = M_2(z_1(t)) \times N_1(z_2(t)) \times P_2(z_3(t))$$
$$h_1(z(t)) = M_2(z_1(t)) \times N_2(z_2(t)) \times P_1(z_3(t))$$
$$h_1(z(t)) = M_2(z_1(t)) \times N_2(z_2(t)) \times P_2(z_3(t))$$

Finally, the nonlinear system (Eq. 1,2,3) will be the sum of 8 linear models interpolated by a nonlinear functions, given as a final result:

$$x(t+1) = \frac{\sum_{i=1}^{r} \mu_i(z(t))\{A_i x(t) + B_i u\}}{\sum_{i=1}^{r} \mu_i(z(t))} \quad (11)$$

And the final outputs of the fuzzy model for the Continuous Fuzzy System are inferred as follows:

$$\dot{x} = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u\} \quad (12)$$

V. SIMULATION RESULTS

In Fig 2, the evolution of the system output are given by the differential equations (Eq.1,2,3) which has been obtained by using the formulation (Eq.10) from the inputs shown in Fig.1.

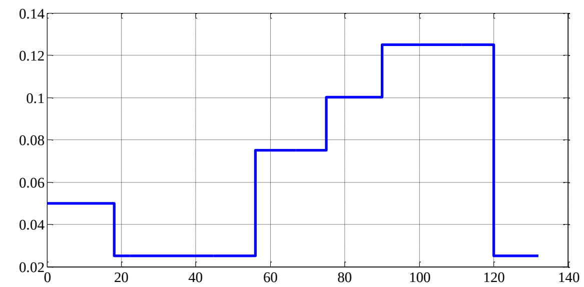
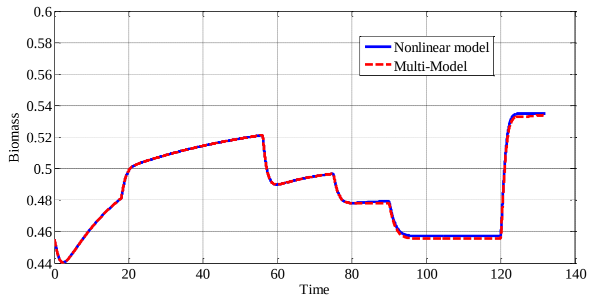


Fig. 1. System Inputs D



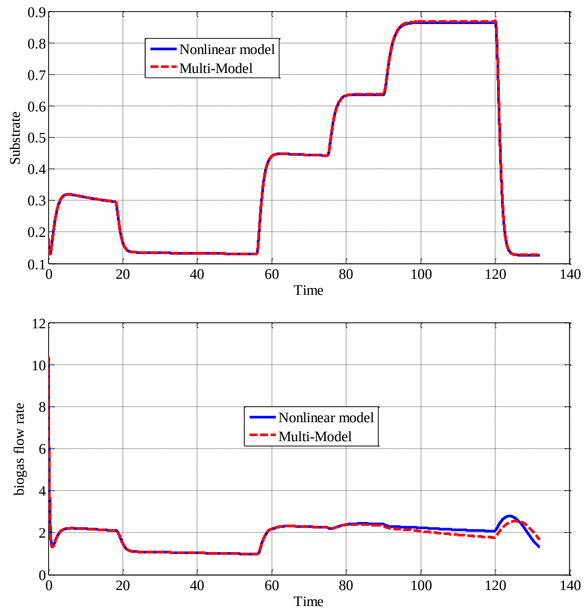


Fig. 2. Outputs of the Nonlinear System and Multi-Model

This T-S fuzzy model can exactly represent the nonlinear system. Figure 2 shows the implementation of the above fuzzy model in Matlab/Simulink. As it is evident in Figure 2, the time responses of the fuzzy model can exactly follow the responses of the original differential equations, which means that the fuzzy model can exactly represent the original system in the pre-specified domains.

VI. Conclusion

In this paper, an application of the T-S method for biotechnological process modeling is presented and detailed. Results show an exact match between the T-S model behavior and the original nonlinear model behavior. However, linear techniques can be applied easily, because of the available tools (linear algebra, differential equations, and differential linear systems, etc.); especially those related to the control algorithms.

References

[1] L. Tan, Deublein D., A. Steinhauser (2008). Biogas from Waste and Renewable Resources.An Introduction, Weinheim, Wiley-VCH.

[2] Simeonov I. (1999). Mathematical Modeling and Parameters Estimation of AnaerobicFermentation Processes, Bioprocess Engrg., 21, 377–381.

[3] Galava H. V., I. Angelidaki, B. K. Ahring (2003). Kinetics and Modeling of Anaerobic Digestion Process, Adv. Biochem. Eng. Biotechnol., 57–93.

[4] Z. Binder, \About a multimodel control methodology, algorithm, multi-processors,implementation and application.," in 8th IFAC World Congress, (Kyoto), pp. 981,986, 1991.

[5] T. TAKAGI and M. SUGENO, \Fuzzy identification of systems and its applications to modeling and control," IEEE transactions on systems, man, and cybernetics, vol. 15,no. 1, pp. 116{132, 1985.

[6] C. Fantuzzi and R. Rovatti, \On the approximation capabilities of the homogeneous Takagi-Sugenomodel," in Fuzzy Systems, 1996., Proceedings of the Fifth IEEE Inter-national Conference on, vol. 2, 1996.

[7] J. Buckley, \Universal fuzzy controllers," Automatica (Journal of IFAC), vol. 28,no. 6, pp. 1245{1248, 1992.

[8] Lyberatos G., I. V. Skiadas (1999). Modelling of Anaerobic Digestion – A Review.Global Nest. Int. J., 1(1), 63-76.

[9] James A. (Ed.) (1978). Mathematical Models in Water Pollution Control, John Wiley and Sons, N.Y.

[10] Dochain D., P. Vanrolleghem (2001). Dynamical Modeling and Estimation in Wastewater Treatment Processes, IWA Publishing, UK.

[11] Murray-Smith R., Johansen T., Multiple model approaches to modeling and control. Taylor and Francis,London,1997.

[12] li N., Li S.Y.,and Xi Y.G.Multi-model predictive control based on the Takagi Sugeno fuzzy models-a case study. Information Science,165:247-263,2004.

[13] Smets I, Verdickt L, Van Impe J, A linear ASM1 based multi- model for activated sludge systems. Mathematical and Computer Modeling of Dynamical Systems, vol.12, pp.489-503,2006.

[14] Takagi, T; Sugeno, M, Fuzzy identification of systems and its application to modelling and control, IEEE Trans.Syst, Man Cybernet 1985,15,166-172./

[15] K. Gasso, Identification des syst`emes dynamiques non-lneaires approchemulti-modèles, Thèse de doctorat,INPL,2000.

[16] K. Tanaka and H.O. Wang, Fuzzy control systems design and analysis: a linear matrix inequality approach,John Wiley and Son Eds,New York, Wiley-Inter science ISBN 0-471-32324-190000, 2001.

[17] Y. Morre, Mise en oeuvre de lois de commande pour les mod`eles flous de type Takagi-Sugeno, Th`ese de doctorat, Universit´e de Valenciennes et du Hainaut-Cambr´esis, 2001.

[18] Wang H.O, Tanaka K, Griffin M, An approach to fuzzy control of nonlinear systems, IEEE Trans. on Fuzzy Systems, vol.4,pp.14-23,1996.

[19] Huang Y., Jadbabaie A., Nonlinear Hinf Control : An enhanced Quasi-LPV Approach, Proc. of the IFAC World Congres, pp. 85-90,1999.

[20] K. Tanaka, \A Theory of Advanced Fuzzy Control," 1994.

[21] Sette Diop1, *, Ivan Simeonov, On the Biomass Specific Growth Rates Estimation for Anaerobic Digestion using Differential Algebraic Techniques. Bioautomation, 13 (3), 47-56, 2009.